Unit 3 Project Wrap-Up

At the Movies

- Investigate one of your favourite movies. Find and record the box office revenues for the first 10 weeks. You may wish to change the time period depending on the availability of data, but try to get about ten successive data points.
- Graph the data.
- Which type of function do you think would best describe the graph? Is one function appropriate or do you think it is more appropriate to use different functions for different parts of the domain?
- Develop a function (or functions) to model the movie's cumulative box office revenue.
- Use your function to predict the cumulative revenue after week 15.
- Discuss whether this model will work for all movies.

Be prepared to present your findings to your classmates.

Cumulative Review, Chapters 7–8

Chapter 7 Exponential Functions

1. Consider the exponential functions $y = 4^x$ and $y = \frac{1}{4}^{x}$.

a) Sketch the graph of each function.

b) Compare the domain, range, intercepts, and equations of the asymptotes.

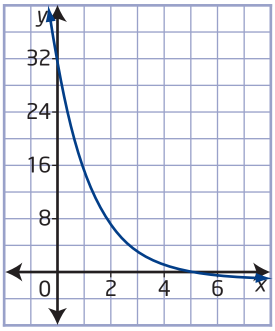
c) Is each function increasing or decreasing? Explain.

2. Match each equation with its graph.

a) $y = 5(2^x) + 1$ **b)** $y = \left(\frac{1}{2}\right)^{x+5}$

c) $y + 1 = 2^{5-x}$ **d)** $y = 5\left(\frac{1}{2}\right)^{-x}$

A



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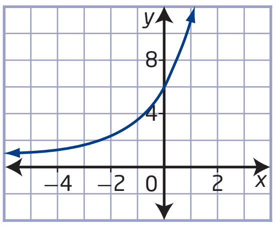

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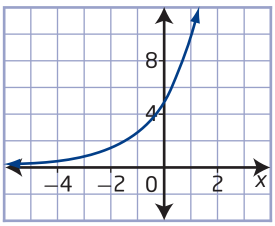

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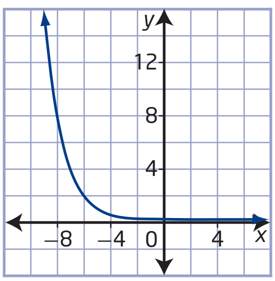


3. The number, B, of bacteria in a culture after t hours is given by $B(t) = 1000\left(2^{\frac{t}{3}}\right)$.

a) How many bacteria were there initially?

b) What is the doubling period, in hours?

c) How many bacteria are present after 24 h?

d) When will there be 128 000 bacteria?

4. The graph of $f(x) = 3^x$ is transformed to obtain the graph of $g(x) = 2(3^{x+4}) + 1$.

a) Describe the transformations.

b) Sketch the graph of $g(x)$.

c) Identify the changes in the domain, range, equations of the asymptotes, and any intercepts due to the transformations.

5. Write the expressions in each pair so that they have the same base.

a) 2^{3x+6} and 8^{x-5} **b)** 27^{4-x} and $\left(\frac{1}{9}\right)^{2x}$

6. Solve for x algebraically.

a) $5 = 2^{x+4} - 3$ **b)** $\frac{25^{x+3}}{625^{x-4}} = 125^{2x+7}$

7. Solve for x graphically. Round your answers to two decimal places.

a) $3(2^{x+1}) = 6^{-x}$

b) $4^{2x} = 3^{x-1} + 5$

8. A pump reduces the air pressure in a tank by 17% each second. Thus, the percent air pressure, p, is given by $p = 100(0.83^t)$, where t is the time, in seconds.

a) Determine the percent air pressure in the tank after 5 s.

b) When will the air pressure be 50% of the starting pressure?

Chapter 8 Logarithmic Functions

9. Express in logarithmic form.

a) $y = 3^x$ **b)** $m = 2^{a+1}$

10. Express in exponential form.

a) $\log_x 3 = 4$ **b)** $\log_a (x + 5) = b$

11. Evaluate.

a) $\log_3 \frac{1}{81}$

b) $\log_2 \sqrt{8} + \frac{1}{3}\log_2 512$

c) $\log_2 (\log_5 \sqrt{5})$

d) 7^k, where $k = \log_7 49$

12. Solve for x.

a) $\log_x 16 = 4$

b) $\log_2 x = 5$

c) $5^{\log_5 x} = \frac{1}{125}$

d) $\log_x (\log_3 \sqrt{27}) = \frac{1}{5}$

13. Describe how the graph of $y = \frac{\log_6 (2x - 8)}{3} + 5$ can be obtained by transforming the graph of $y = \log_6 x$.

14. Determine the equation of the transformed image of the logarithmic function $y = \log x$ after each set of transformations is applied.

a) a vertical stretch about the x-axis by a factor of 3 and a horizontal translation of 5 units left

b) a horizontal stretch about the y-axis by a factor of $\frac{1}{2}$, a reflection in the x-axis, and a vertical translation of 2 units down

15. The pH of a solution is defined as $\text{pH} = -\log [H^+]$, where $[H^+]$ is the hydrogen ion concentration, in moles per litre. The pH of a soil solution indicates the nutrients, such as nitrogen and potassium, that plants need in specific amounts to grow.

a) Alfalfa grows best in soils with a pH of 6.2 to 7.8. Determine the range of the concentration of hydrogen ions that is best for alfalfa.

b) When the pH of the soil solution is above 5.5, nitrogen is made available to plants. If the concentration of hydrogen ions is 3.0×10^{-6} mol/L, is nitrogen available?

16. Write each expression as a single logarithm in simplest form. State any restrictions on the variables.

a) $2 \log m - (\log \sqrt{n} + 3 \log p)$

b) $\frac{1}{3}(\log_a x - \log_a \sqrt{x}) + \log_a 3x^2$

c) $2 \log (x + 1) + \log (x - 1) - \log (x^2 - 1)$

d) $\log_2 27^x - \log_2 3^x$

17. Zack attempts to solve a logarithmic equation as shown. Identify and describe any errors, and then correctly solve the equation.

$\log_3 (x - 4)^2 = 4$
$3^4 = (x - 4)^2$
$81 = x^2 - 8x + 16$
$0 = x^2 - 8x - 65$
$x = -13$ or $x = 5$

18. Determine the value of x. Round your answers to two decimal places if necessary.

a) $4^{2x+1} = 9(4^{1-x})$

b) $\log_3 x + 3 \log_3 x^2 = 14$

c) $\log (2x - 3) = \log (4x - 3) - \log x$

d) $\log_2 x + \log_2 (x + 6) = 4$

19. The Richter magnitude, M, of an earthquake is related to the energy, E, in joules, released by the earthquake according to the equation $\log E = 4.4 + 1.4M$.

a) Determine the energy for earthquakes with magnitudes 4 and 5.

b) For each increase in M of 1, by what factor does E change?

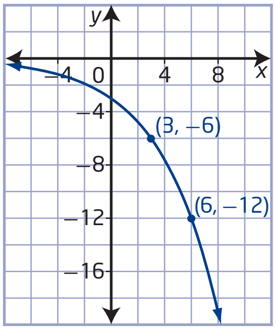
20. At the end of each quarter year, Aaron makes a \$625 payment into a mutual fund that earns an annual percentage rate of 6%, compounded quarterly. The future value, FV, of Aaron's investment is $FV = \frac{R[(1 + i)^n - 1]}{i}$, where n is the number of equal periodic payments of R dollars, and i is the interest rate per compounding period expressed as a decimal. After how long will Aaron's investment be worth \$1 000 000?

Unit 3 Test

Multiple Choice

For #1 to #7, select the best answer.

1. The graph of the function $y = a(2^{bx})$ is shown.



The value of a is

A 3

B $\frac{1}{3}$

C $-\frac{1}{3}$

D −3

2. The graph of the function $y = b^x$, $b > 1$, is transformed to $y = 3(b^{x+1}) - 2$. The characteristics of the function that change are

A the domain and the range

B the range, the x-intercept, and the y-intercept

C the domain, the x-intercept, and the y-intercept

D the domain, the range, the x-intercept, and the y-intercept

3. The half-life of carbon-14 is 5730 years. If a bone has lost 40% of its carbon-14, then an equation that can be used to determine its age is

A $60 = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$

B $60 = 100\left(\frac{1}{2}\right)^{\frac{5730}{t}}$

C $40 = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$

D $40 = 100\left(\frac{1}{2}\right)^{\frac{5730}{t}}$

4. Which of the following is an equivalent form for $2x = \log_3 (y - 1)$?

A $y = 3^{2x} - 1$

B $y = 3^{2x+1}$

C $y = 9^x + 1$

D $y = 9^{x+1}$

5. The domain of $f(x) = -\log_2 (x + 3)$ is

A $\{x \mid x > -3, x \in \mathrm{R}\}$

B $\{x \mid x \geq -3, x \in \mathrm{R}\}$

C $\{x \mid x < 3, x \in \mathrm{R}\}$

D $\{x \mid x \in \mathrm{R}\}$

6. If $\log_2 5 = x$, then $\log_2 \sqrt[4]{25^3}$ is equivalent to

A $\frac{3x}{2}$

B $\frac{3x}{8}$

C $x^{\frac{3}{2}}$

D $x^{\frac{3}{8}}$

7. If $\log_4 16 = x + 2y$ and $\log 0.0001 = x - y$, then the value of y is

A −2

B $-\frac{1}{2}$

C $\frac{1}{2}$

D 2

Numerical Response

Copy and complete the statements in #8 to #12.

8. The graph of the function $f(x) = \left(\frac{1}{4}\right)^x$ is transformed by a vertical stretch about the x-axis by a factor of 2, a reflection about the x-axis, and a horizontal translation of 3 units right. The equation of the transformed function is ■.

9. The quotient $\frac{9^{\frac{1}{2}}}{27^{\frac{2}{3}}}$ expressed as a single power of 3 is ■.

10. The point P(2, 1) is on the graph of the logarithmic function $y = \log_2 x$. When the function is reflected in the x-axis and translated 1 unit down, the coordinates of the image of P are ■.

11. The solution to the equation $\log 10^x = 0.001$ is ■.

12. Evaluating $\log_5 40 - 3 \log_5 10$ results in ■.

Written Response

13. Consider $f(x) = 3^{-x} - 2$.
    a) Sketch the graph of the function.
    b) State the domain and the range.
    c) Determine the zeros of $f(x)$, to one decimal place.

14. Solve for x and verify your solution.
    a) $9^{\frac{1}{4}}\left(\frac{1}{3}\right)^{\frac{x}{2}} = \sqrt[3]{27^4}$
    b) $5(2^{x-1}) = 10^{2x-3}$

15. Let $f(x) = 1 - \log (x - 2)$.
    a) Determine the domain, range, and equations of the asymptotes of $f(x)$.
    b) Determine the equation of $f^{-1}(x)$.
    c) Determine the y-intercepts of $f^{-1}(x)$.

16. Solve for x algebraically.
    a) $\log 4 = \log x + \log (13 - 3x)$
    b) $\log_3 (3x + 6) - \log_3 (x - 4) = 2$

17. The following shows how Giovanni attempted to solve the equation $2(3^x) = 8$. Identify, describe, and correct his errors.

$$2(3^x) = 8$$
$$6^x = 8$$
$$\log 6^x = \log 8$$
$$x \log 6 = \log 8$$
$$x = \frac{\log 8}{\log 6}$$
$$x = \log 8 - \log 6$$
$$x \approx 0.12$$

The solution is $x \approx 0.12$.

18. The Richter magnitude, M, of an earthquake is defined as $M = \log \left(\frac{A}{A_0}\right)$, where A is the amplitude of the ground motion and A_0 is the amplitude, corrected for the distance to the actual earthquake, that would be expected for a standard earthquake. An earthquake near Tofino, British Columbia, measures 5.6 on the Richter scale. An aftershock is $\frac{1}{4}$ the amplitude of the original earthquake. Determine the magnitude of the aftershock on the Richter scale, to the nearest tenth.

19. The world population was approximately 6 billion in 2000. Assume that the population grows at a rate of 1.3% per year.
    a) Write an equation to represent the population of the world.
    b) When will the population reach at least 10 billion?

20. To save for a new highway tractor, a truck company deposits \$11 500 at the end of every 6 months into an account with an annual percentage rate of 5%, compounded semi-annually. Determine the number of deposits needed so that the account has at least \$150 000. Use the formula $FV = \frac{R[(1 + i)^n - 1]}{i}$, where FV is the future value, n is the number of equal periodic payments of R dollars, and i is the interest rate per compounding period expressed as a decimal.